Abstract: In this paper, we present a framework for evaluating commercially developed mathematics programs. This framework is based on principles of instructional design derived from a Direct Instruction approach to education. Given the role that instructional programs play in the classroom, especially for teachers who have not been well prepared to teach mathematics, the quality of commercially developed mathematics programs needs to be closely scrutinized. The Mathematics Curriculum Evaluation Framework outlined here is designed to help teachers evaluate mathematics programs to select new programs or modify the mathematics programs available to them. While this framework is not exhaustive, it will give teachers a focus for their curriculum evaluation efforts and will also help teachers identify areas in mathematics programs that can be easily modified. Finally, this framework can serve as the first stage in the development of a reliable and valid curriculum evaluation instrument for determining the quality of commercially developed mathematics programs.

According to the 2003 National Assessment of Educational Progress (NAEP), only 32% of fourth-grade students and 29% of eighth-grade students scored at the proficient level in mathematics (National Center for Educational Statistics, 2003). According to the National Assessment Governing Board, students reaching proficiency have demonstrated competency over challenging mathematics content including mathematics knowledge, application to real-world situations, and analytical skills. Similarly, Schmidt, Houang, and Cogan (2002) reported that by the end of high school, students from the United States performed near the bottom of the international distribution in the Third International Mathematics and Science Study (TIMSS), the most extensive comparative study of math and science achievement and curriculum to date. Furthermore, research suggests that many students who are learning disabled lag behind their typically achieving peers in the area of mathematics (Carnine, Jones, & Dixon, 1994). These reports of poor student performance in both general and special education have compelled educators to examine the mathematics instruction in this country more closely.

The National Council of Teachers of Mathematics (NCTM), in an attempt to ensure the highest quality mathematics instruction for all students, outlined several principles of effective mathematics instruction. These broad principles address several important areas including curriculum and teaching (NCTM, 2000). For example, the NCTM principles highlight the need for well designed curricula as well as the need for quality teacher preparation that provides teachers with core mathematics knowledge. Ma (1999), in an extensive study of Chinese
and American teachers’ knowledge of mathematics, confirmed that the knowledge of most American elementary mathematics teachers was not nearly as robust as that of Chinese educators she interviewed. Schmidt et al. (2002), in their analysis of the TIMSS data, also addressed issues of curriculum and teaching. They stated, “American students and teachers are greatly disadvantaged by our country’s lack of a common coherent curriculum and the text, materials, and training that match it” (p. 10).

Historically, few experimental studies have investigated specific instructional methods or curricular components in the area of mathematics (Gersten, 2002). An exception to this dearth of research on mathematics methods and materials comes from Direct Instruction. As reported by Adams and Engelmann (1996) and Przychodzin, Marchand-Martella, Martella, and Azim (2004), using a Direct Instruction approach to teach mathematics results in increased achievement when compared to other instructional approaches.

One of the ways in which Direct Instruction differs from most educational approaches is in the application of precise principles of instructional design to curriculum development (Carnine, 1997; Dixon, 1994; Engelmann & Carnine, 1991; Harniss, Stein, & Carnine, 2002). These design principles have been applied to a wide range of curricular areas including reading, writing, spelling, and critical thinking. The purpose of this paper is to present a framework for evaluating mathematics programs based on these principles. Educators involved in adopting a mathematics program can use this framework to assist them in selecting well designed commercially developed materials. In addition, educators can use this framework for evaluating and modifying their current mathematics programs to better meet the instructional needs of their students.

The Curriculum Adoption Process

The process of curriculum adoption is critical to the selection of high quality instructional materials. Therefore, educators should not only employ a systematic framework for evaluating those materials but should also conduct the adoption process in an equally systematic manner. Stein, Stuen, Carnine, and Long (2001) described some of the critical features of a systematic adoption process for the selection of reading programs. Not surprisingly, these features should also be present when selecting mathematics programs. Although a thorough discussion of the adoption process is beyond the scope of this article, we have highlighted the features from Stein et al. that we believe are essential to conducting an effective curriculum adoption in the area of mathematics. These features include: time allocation, committee responsibilities, and the screening process.

Time Allocation

A major consideration in the adoption of curriculum materials is allocating sufficient time for the screening and evaluation of those materials. Many curriculum adoption committees work only after school for brief periods of time. However, meaningful and thorough examination of instructional materials requires large blocks of uninterrupted time. Therefore, committee members must be given adequate release time to review the materials and discuss their findings with their colleagues. Stein et al. (2001) offer an example of a timeline for curriculum adoption that allocates approximately 15 release days for teachers on the adoption committee. During those days teachers review research, generate screening and evaluation criteria, screen all submitted programs, thoroughly evaluate three to four of those programs, deliberate, and then select a program.
Committee Responsibilities
Commonly, teachers in schools or districts are given the opportunity to vote on the selection of a mathematics program from a short list generated by an adoption committee. Given that the adoption committee members are given adequate time to evaluate the mathematics programs thoroughly, the final selection of an instructional program should rest with the adoption committee. To feel comfortable with a committee decision, however, most teachers need to be kept informed at all stages of the adoption process. Therefore, adoption committee members must communicate regularly and effectively with the groups they represent.

Members of mathematics curriculum adoption committees are often selected based on seniority and knowledge of mathematics. However, additional selection factors also should be considered when forming adoption committees. Committees should include individuals representing a range of grade levels, those representing both special and general education students, and those with excellent communication skills.

Screening Process
To expedite the task of evaluating instructional programs, we recommend that the adoption committee first screen all the programs submitted for consideration. Given that evaluating programs thoroughly requires a substantial time commitment, screening is recommended to reduce the number of programs that the committee must eventually evaluate.

The first step in the screening process is to determine the criteria that will be used. Table 1 illustrates an example of criteria that we believe will facilitate the screening process. The criteria include questions that address three important areas: (a) General Instructional Approach, (b) Evidence of Effectiveness, and (c) Critical Content. The questions under General Instructional Approach direct evaluators to ascertain the program’s theoretical approach—that is, whether the program represents an explicit or

| Table 1 |

Mathematics Curriculum Evaluation Framework: Screening Criteria

A. General Instructioonal Approach
1. Does the program contain explicit instruction, i.e., steps in the strategies are clearly identified for both teachers and students?
   OR
2. Does the program represent a constructivist approach, i.e., student discovery and exploration is emphasized?

B. Evidence of Effectiveness
1. Is there published evidence of the effectiveness of the program?
2. Is there evidence that the program has been field tested with large groups of students?

C. Critical Content
1. Are the steps in the selected strategies explicitly identified in the program?
2. Does the instruction follow a logical sequence?
3. Are there sufficient practice opportunities for mastery distributed across the grade level?
direct approach, a constructivist approach, or another approach to the teaching of mathematics. The questions under Evidence of Effectiveness direct evaluators to determine if the program has been systematically evaluated in controlled research studies that are subsequently published in the research literature. In addition to published research, evidence of whether the program has been field-tested is considered in this section as well.

The questions under Critical Content can be used to compare how different programs teach important skills or concepts. We recommend that for screening purposes, evaluators compare two skills from each program in two grade levels (e.g., one primary grade, one intermediate grade). By comparing how programs teach these two skills, evaluators can get a sense of the overall program design. The questions in this section were selected for screening purposes from a more comprehensive set of evaluation questions that appear in the Mathematics Evaluation Framework outlined later in this article.

Mathematics Curriculum Evaluation Framework

On the following pages we describe a curriculum evaluation framework for examining commercially developed mathematics programs. We have developed a form that may be used to assist teachers in using the framework to evaluate materials (see Table 2). The framework contains four sections: General Program Design, Instructional Strategy Design, Teaching Procedures, and Assessment. This form has three columns: Evaluation Criteria, Comments, and Examples. The Examples column is included to provide specific references to examples (i.e., page numbers from the program) that illustrate and support the evaluator’s comments. These examples are necessary to engage in an objective discussion about program quality with other committee members.

I. General Program Design

The purpose of the General Program Design criteria is to provide evaluators with an overview of the program’s goals/objectives and degree of program coherence. The questions in this section address design features relevant to all levels in a given program. To answer these questions, we recommend that evaluators examine the scope and sequence of each level as well as examine sample lessons in a primary and intermediate level. Program coherence in this framework refers to the extent to which the content of the program is integrated within and across grade levels and to the balance and integration of computation and problem solving.

A. Program Goals/Objectives

1) Are the “big ideas” in the program obvious? According to principles of Direct Instruction, well designed mathematics programs are organized around major principles (i.e., big ideas or goals) that are applicable in many situations and contexts (e.g., place value, equivalence, number sense). Although all programs will contain instruction on big ideas, evaluators should determine the extent to which the big ideas are well articulated and obvious by looking at the program’s scope and sequence. A significant amount of instructional time should be devoted to these concepts, and evaluators should be able to determine the extent to which the concepts appear in many related contexts.

2) Are objectives stated as observable behaviors? The objectives in a program help teachers determine exactly what students should be able to do as a result of the instruction provided. Not only should objectives be aligned with the instruction but they should also be aligned with the program assessment procedures. The objectives should contain a statement of a measurable behavior. Many programs contain objectives that describe teacher behavior and not student behavior. For example, we found objectives similar to this one in several mathe-
Table 2  
*Mathematics Curriculum Evaluation Framework*

Evaluator(s) ____________________________________________ Grade Level___________
Program/Publisher/Year____________________________________ Date_________________

<table>
<thead>
<tr>
<th>Evaluation Criteria</th>
<th>Comments</th>
<th>Examples</th>
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<tr>
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<td>1) Does the program use a strand or spiral design?</td>
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<td>2) Is there a balance between computation instruction and problem-solving instruction?</td>
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<td><strong>II. Instructional Strategy Design</strong></td>
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<td>1) Are the steps in the strategy explicitly identified in the program?</td>
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<td>B. Sequence and Integration</td>
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<td>1) Are the necessary component skills (preskills) taught prior to introducing the strategy?</td>
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*Mathematics Curriculum Evaluation Framework*

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<tbody>
<tr>
<td>C. Examples</td>
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<tr>
<td>1) Is there a sufficient number of <strong>practice examples</strong> for initial mastery?</td>
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<td>2) Are there opportunities for <strong>discrimination practice</strong>?</td>
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<tr>
<td>3) Does the program provide opportunities for <strong>cumulative review</strong> of previously introduced skills?</td>
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</table>

**III. Teaching Procedures**

**A. Scaffolded Instruction**

1) Is teacher modeling specified?  
2) Is teacher assistance gradually faded?  
3) Does the program recommend specific correction procedures?

**IV. Assessment**

**A. Assessment and Instruction Link**

1) Does the program contain placement tests?  
2) Do the program assessments contain recommendations for acceleration and remediation?  
3) Are the program assessments carefully aligned with instruction?
matics programs: “Review telling time.” Note that the objective identifies what the teacher does but not what the students do. Alternatively this time telling objective is stated as a measurable behavior: “Students will express time as minutes after the hour.” Teachers would have little difficulty assessing whether students met this objective.

B. Program Coherence

1) Does the program use a strand or spiral design? As Snider (2004) notes, programs using a strand design teach fewer topics over a long period of time with the goal of student mastery. Programs using a spiral design, in contrast, present a large number of topics for a short period of time with relatively little depth or integration with the goal of exposure. Evaluators can use the scope and sequence of individual grade levels and the overviews of the programs provided by the publishers to ascertain the overall approach to the design of the content. For more detailed information on strand versus spiral design, see Snider, this issue.

2) Is there a balance between computation instruction and problem-solving instruction? Wu (1999) was among the first to identify the false dichotomy between computation and problem solving. He wrote that a common mathematics misconception held by many educators and the general public is, “… that the demand for precision and fluency in the execution of basic skills in school mathematics runs counter to the acquisition of conceptual understanding” (p. 14). Wu articulated concern over the practice of increasing instruction in abstract conceptual understanding while decreasing (or even eliminating) instruction in basic skills. As a mathematician, he fully realizes that understanding mathematics at the deepest level requires instruction that addresses both computation and problem solving and that the two processes are not mutually exclusive. Therefore, we recommend that evaluators carefully examine the scope and sequence of each program to determine the extent to which it provides an instructional balance between computation and problem solving.

II. Instructional Strategy Design

The criteria for Instructional Strategy Design were developed to help evaluators determine how critical content is taught in each of the programs. We recommend that grade-level committees select three to four different skills or concepts for each grade level, preferably ones that have been identified as big ideas, and use the criteria specified in Table 2 to examine how systematically those skills or concepts are taught.

To best evaluate the instructional strategies, we recommend that evaluators conduct a skill trace for each of the core skills or concepts they choose to evaluate. A skill trace involves locating every instance where the target strategy appears in the program. The skill trace helps evaluators isolate the instructional strategy to best evaluate its explicitness and generalizability. The skill trace also allows evaluators to determine whether component skills are identified and taught prior to the introduction of the strategy. Finally, a skill trace will also provide evidence of the degree to which the strategies are integrated with each other.

A. Strategy

1) Are the steps in the strategy explicitly identified in the program? To determine the explicitness of a strategy, evaluators should examine where in the program the strategy is first introduced and determine whether the program clearly articulates the steps that students are to follow in using the strategy to solve a problem (either a computation problem or word problem). The following is an example of a strategy for solving word problems that would not be considered explicit: “1. Decide what to do. 2. Do it. 3. Does my answer make sense?” (The University of Chicago School Mathematics Project, 1995, p. 44). Although the steps in the strategy are clearly articulated, the above strategy is not considered to be explicit because the cognitive
processes required by students to solve the problem are not overtly taught.

In contrast, an explicit direct instruction strategy for solving classification word problems is outlined in Figure 1 (Stein, Silbert, & Carnine, 1997). Before students are introduced to this strategy, they are taught several component skills including the language skill of identifying class names for groups of objects (e.g., saws, hammers, and screwdrivers are all tools) and how to use a fact number family strategy to solve addition and subtraction facts (e.g., 2, 4, 6 comprise a fact family used to generate $2 + 4 = 6$, $4 + 2 = 6$, $6 - 2 = 4$, $6 - 4 = 2$). Also, prior to being introduced to this strategy, students would be familiar with the graphic conventions of big and little boxes. Figure 1 provides an example of an explicit strategy for teaching students how to solve a complex type of word problem.

2) Is the strategy of intermediate generalizability—not too narrow or too broad? A well designed strategy results in the greatest number of students correctly solving the greatest number of problems. The strategy should reliably lead to the solution of the problem for all students. The steps in the general word problem strategy mentioned above (“Decide what to do. Do it. Does my answer make sense?”) are so broad that only students who already know how to solve the problem will correctly answer the question.

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**Figure 1**

*Solving classification word problems.*

<table>
<thead>
<tr>
<th>There are 8 children. Three are boys. How many are girls?</th>
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</table>

1. Read the problem. The problem talks about children, boys, and girls. Which is the big class?

2. If children is the big class then children is the total number. Write children on the line under the total box.

3. Does the problem tell how many children? So the total number is given. What is the total number? Write 8 in the box for Total number.

4. Now we write the values for boys and girls in the boxes over the arrow. How many boys? Write 3 in the first box. We don’t know how many girls so we don’t write anything in the other box.

5. Is the total number given? So what do you have to do to work the problem? I start with 8 children and subtract 3 boys to find out how many girls. Write the equation and figure out the answer.

6. If there are 8 children and 3 are boys, how many are girls?
Similarly, teachers do not want to spend valuable instructional time teaching a narrow strategy that is of limited use. Teaching students a shortcut for finding 1/3 of 9 by teaching them to divide 9 by 3 only works when the fraction in the problem has a 1 in the numerator. Low performing students are likely to overgeneralize this strategy when asked to find 2/3 of 9 and answer incorrectly.

We feel strongly that the quality of the instructional strategies included in mathematics programs should be a defining component in the evaluation of these programs. A practical approach to the evaluation of the strategies is for evaluators to assume the characteristics of naïve learners. Evaluators should pretend that they do not already know how to solve the problem and follow the steps in the strategy as specified in the teacher’s manual. By doing this, evaluators can determine whether the steps in the strategy are explicit and useful.

B. Sequence and Integration

1) Are the necessary component skills (preskills) taught prior to introducing the strategy? Often mathematics programs introduce the component skills and the new strategy simultaneously. For example, a program will introduce estimation at the same time that it introduces long division, a skill requiring the use of estimation. Most students need time to master the component skills prior to being introduced to a strategy that requires the application of that component skill. Students should have mastered estimation prior to the introduction of long division.

2) Does the program strategically integrate the new strategy with previously introduced strategies and related skills? Although we acknowledge the advantages of introducing new strategies in isolation, we also understand that students must learn the relationship among strategies to understand mathematics concepts fully. Therefore, one of the features of a well designed program is the strategic integration of mathematics strategies throughout the program. One way to determine the integration is to examine the skill trace for evidence that the newly taught strategy has been integrated with related previously taught strategies. Ideally, this integration would occur at the end of the instructional sequence after students have demonstrated mastery of the strategy in isolation. For example, well designed programs carefully integrate long division into word problems after students have demonstrated mastery on solving long division problems in isolation.

C. Examples

1) Is there a sufficient number of practice examples for initial mastery? Technically, this question can only be answered using information about student performance (i.e., Did students master this skill with the number of practice examples available?). Therefore, evaluators should compare programs with respect to the number of examples provided and err in selecting programs with more rather than fewer examples. Reducing the number of practice examples presented is far easier than creating additional examples for students who need more practice.

2) Does the program contain opportunities for discrimination practice? Discrimination practice refers to including a set of practice examples that requires students to determine when to apply a strategy and when not to. For example, after presenting subtraction with regrouping, the program should give students the opportunity to practice regrouping using a set of examples in which some problems require regrouping and some do not. Without that practice, some students will try to apply the regrouping strategy to any multidigit subtraction problem they encounter.

3) Does the program provide opportunities for cumulative review? Cumulative review refers to the notion that all strategies taught should be systematically reviewed throughout the
III. Teaching Procedures

The Teaching Procedures criteria focus on how well the program supports the teacher by providing specific teacher instructions. Traditionally, mathematics programs suggest teachers demonstrate a strategy with a couple of problems, then direct teachers to have students complete a number of problems independently (Hamiss, Carnine, Silbert, & Dixon, 2002). In contrast, one of the most critical teaching procedures derived from Direct Instruction design principles is the use of scaffolded instruction. Scaffolded instruction is the use of scaffolding to assist students with the learning process. It begins with the teacher modeling a strategy to help students achieve mastery. The questions in this section will determine whether the program includes procedures that require the teacher to model the steps in the instructional strategy. It should be noted that some programs may suggest teacher modeling but may not explicitly provide the steps for the strategy in the teacher’s manual. scaffolded instruction provides temporary support to students as they begin to apply their new strategies. Programs may provide a series of questions for the teacher to use in guiding students through the steps necessary to complete problems. As students become more proficient, the teacher asks fewer guiding questions as students complete problems. Alternatively, the program may supply graphic support for students in applying their new strategy. For example, a graphic organizer may prompt students to find the common denominators prior to adding or subtracting fractions. Scaffolded instruction provides teachers with the support necessary to ensure that students solve mathematics problems with fewer errors as they become more independent.

IV. Assessment

Finally, the assessment criteria address the quality of the placement and evaluation procedures recommended in the programs with respect to the link between assessment and instruction. Evaluators examine the teacher’s manuals as well as any supplementary assessment materials to answer the questions in this section. We suggest that evaluators first determine the alignment of the assessment materials to the content. Each section contains questions that are intended to guide evaluators in examining the materials. The most important consideration is the program’s specific correction procedures.
mine whether programs contain a placement test with alternative placement options so that students can be placed at appropriate levels of the program. Next, evaluators should determine if recommendations for acceleration and remediation are provided based on the program assessment results. Finally, evaluators establish the extent to which program assessments are aligned with instruction. This alignment is necessary for teachers to make informed instructional decisions regarding student progress and mastery of the content.

A. Assessment and Instruction Link

1) Does the program contain placement tests? Program placement tests provide important information regarding the appropriateness of the program placement for individual students. The placement tests should provide teachers with information to determine if the students have the background knowledge and skills required for entrance into a given level of the program. Additionally, the tests could provide multiple entry points for students who have already mastered some of the program content.

2) Do the program assessments contain recommendations for acceleration and remediation? Related to issues of assessment/instruction alignment is the question of whether the program provides specific recommendations for acceleration or remediation based on student performance. Ideally, the program assessments would help teachers identify those students who need more or less assistance in mastering the content so that instruction could be differentiated appropriately.

3) Are the program assessments carefully aligned with instruction? Since the purpose of the in-program assessments is to help teachers make more informed instructional decisions, the assessments must be aligned with program content. The assessment items should test both newly taught skills and concepts as well as previously introduced content.

Conclusion

Teachers of elementary mathematics need well designed programs to teach their students more effectively. The Mathematics Curriculum Evaluation Framework outlined in this paper is designed to help teachers screen mathematics instructional programs and evaluate the extent to which the programs incorporate instructional design features that appear to be related to student achievement. While little research on these features in isolation is available, research on commercially developed Direct Instruction mathematics programs suggests that these features contribute to increased mathematics achievement.

Clearly, systematic research is necessary to develop reliable and valid criteria and objective procedures for the analysis of commercially developed mathematics programs. Currently, there are no reliable and valid evaluation instruments for analyzing these programs. The Mathematics Curriculum Evaluation Framework presented here outlines criteria that may be included in such an instrument. Educators interested in pursuing the development of these instruments will need to validate those criteria, develop objective evaluation procedures, and determine reliability. Moreover, educators will need to design the evaluation process with practitioners in mind. That is, evaluation instruments will need to be easy to use and provide information that is useful to the consumer. Until a reliable and valid curriculum evaluation instrument is designed, we suggest that using this framework will result in a much more thorough examination of the quality of a commercially developed mathematics program.

References


